On Small Longitudinal Material Waves accompanying Light Waves.

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All experiments on the pressure of light agree in showing that there is a flow of momentum along the beam. This flow is manifested as a force on matter wherever there is a change of medium. When the light is absorbed, the momentum is absorbed by matter. When the beam is shifted parallel to itself there is a torque on the matter effecting the shift. The momentum would therefore appear to be carried by the matter and not merely by the ether. Though there is an obvious difficulty in accepting this view when the density of the matter is so small as it is in interplanetary space, it appears to be worth while to follow out the consequences of the supposition that the force equivalent to the rate of flow of momentum across a plane perpendicular to a beam of light acts upon the matter bounded by the plane. This rate of flow per square centimetre is equal to the energy density or energy per cubic centimetre in the beam. Of course, in experiments, only the average of the rate of flow during many seconds and the average energy per cubic centimetre in a length of beam of millions of miles is actually measured. But on the electromagnetic theory of light which suggested the experiments and which gives the right value for the pressure, this pressure is equal to the energy density at every point of a single wave.

Let us suppose that we have a train of plane polarised electromagnetic waves of sine form, the magnetic intensity being given by

$$\mathrm{H} = \mathrm{H}_1 \sin \frac{2\pi}{\lambda}(x - vt),$$

where H_1 is the amplitude of H. The magnetic energy per cubic centimetre at any point is $\mu\mathrm{H}^2/8\pi$, and as the electric energy is equal at each point to the magnetic energy, the total energy is $\mu\mathrm{H}^2/4\pi$.

The energy per unit volume is $\int_0^1 \frac{\mu\mathrm{H}^2}{4\pi}\,dx = \mu\mathrm{H}_1^2/8\pi$.

The pressure p across a transverse surface is

$$p = \mu\mathrm{H}^2/4\pi = \frac{\mu\mathrm{H}_1^2}{4\pi}\sin^2\frac{2\pi}{\lambda}(x - vt)$$

$$= \frac{\mu\mathrm{H}_1^2}{8\pi}\left\{1 - \cos\frac{4\pi}{\lambda}(x - vt)\right\}.$$

The force on an element of length dx is

$$-\frac{dp}{dx}dx = -\frac{\mu H_1^2}{8\pi}\frac{4\pi}{\lambda}\sin\frac{4\pi}{\lambda}(x-vt)\,dx$$

$$= -\frac{\mu H_1^2}{2\lambda}\sin\frac{4\pi}{\lambda}(x-vt).$$

If ξ is the linear longitudinal displacement of the element there will be a force due to elastic change of volume

$$q\frac{d^2\xi}{dx^2}dx,$$

where q is the elastic constant for compression or extension.

If ρ is the density of the material, the equation of motion is

$$\rho\frac{d^2\xi}{dt} = q\cdot\frac{d^2\xi}{dx^2} - \frac{\mu H_1^2}{2\lambda}\sin\frac{4\pi}{\lambda}(x-vt).$$

Assume $$\xi = A\sin\frac{4\pi}{\lambda}(x-vt-\epsilon).$$

Then, substituting,

$$\left(\rho A\cdot\frac{16\pi^2}{\lambda^2}v^2 - qA\frac{16\pi^2}{\lambda^2}\right)\sin\frac{4\pi}{\lambda}(x-vt-\epsilon) = \frac{\mu H^2}{2\lambda}\sin\frac{4\pi}{\lambda}(x-vt).$$

Putting $x=0$ and $t=0$, we see that $\epsilon=0$. Putting $q=\rho u^2$, where u is the velocity of free elastic waves of the q type, and assuming that the longitudinal waves are forced waves, keeping exact time with the waves of light, we have

$$A = \frac{\lambda\mu H_1^2}{32\pi^2\rho(v^2-u^2)}.$$

As u/v is negligible for all ordinary matter,

$$\xi = \frac{\lambda\mu H_1^2}{32\pi^2\rho v^2}\sin\frac{4\pi}{\lambda}(x-vt)$$

$$\dot{\xi} = \frac{\mu H_1^2}{8\pi\rho v}\cos\frac{4\pi}{\lambda}(x-vt).$$

The potential energy in these waves is negligible in comparison with the kinetic. We have then

$$\text{Energy per unit volume} = \tfrac{1}{2}\int_0^1 \rho\dot{\xi}^2 dx = \frac{\mu^2 H_1^4}{256\pi^2\rho v^2}.$$

As the electromagnetic energy per unit volume is $\mu H_1^2/8\pi$,

$$\frac{\text{Energy in longitudinal waves}}{\text{Electromagnetic energy}} = \frac{\mu H_1^2}{32\pi\rho v^2} = \frac{1}{8}\frac{\mu H_1^2}{8\pi}\bigg/\frac{\rho v^2}{2},$$

which is one-eighth of the electromagnetic energy divided by the energy

which the matter would have if it were moving with the velocity of light in that matter.

This shows how infinitesimal is the fraction of the energy of the beam which is located in these waves of compression of the material.

The fraction is proportional to the intensity of the beam.

As an example, take a beam of the intensity of full sunlight just outside the earth's atmosphere, in which the energy flow is about $1 \cdot 4 \times 10^6$ ergs/sec. The energy density $\mu H_1^2/8\pi$ is therefore $1 \cdot 4 \times 10^6 \div v$. Put $v = 3 \times 10^{10}/n$, where n is the refractive index. The fraction is

$$\frac{1}{4} \cdot \frac{1 \cdot 4 \times 10^6}{27 \times 10^{30}} \frac{n^3}{\rho}, \text{ or about } 1 \cdot 25 \times 10^{-26} n^3/\rho.$$

At the surface of the sun it would be about 40,000 times as much, say, $5 \times 10^{-22} n^3/\rho$.

It is interesting to note that if a beam of light is incident on any reflecting or absorbing surface and if the pressure of light is periodic with the waves it must give rise to ordinary elastic waves in the material of frequency double that of the light waves.

The Properties of Colloidal Systems.—II. On Adsorption as Preliminary to Chemical Reaction.

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